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Sola scriptura

Sola fide

Sola gratia

Solus Christus

Soli Deo gloria!

Tangerang, July 5, 2023



(Putu Merta Yasa)

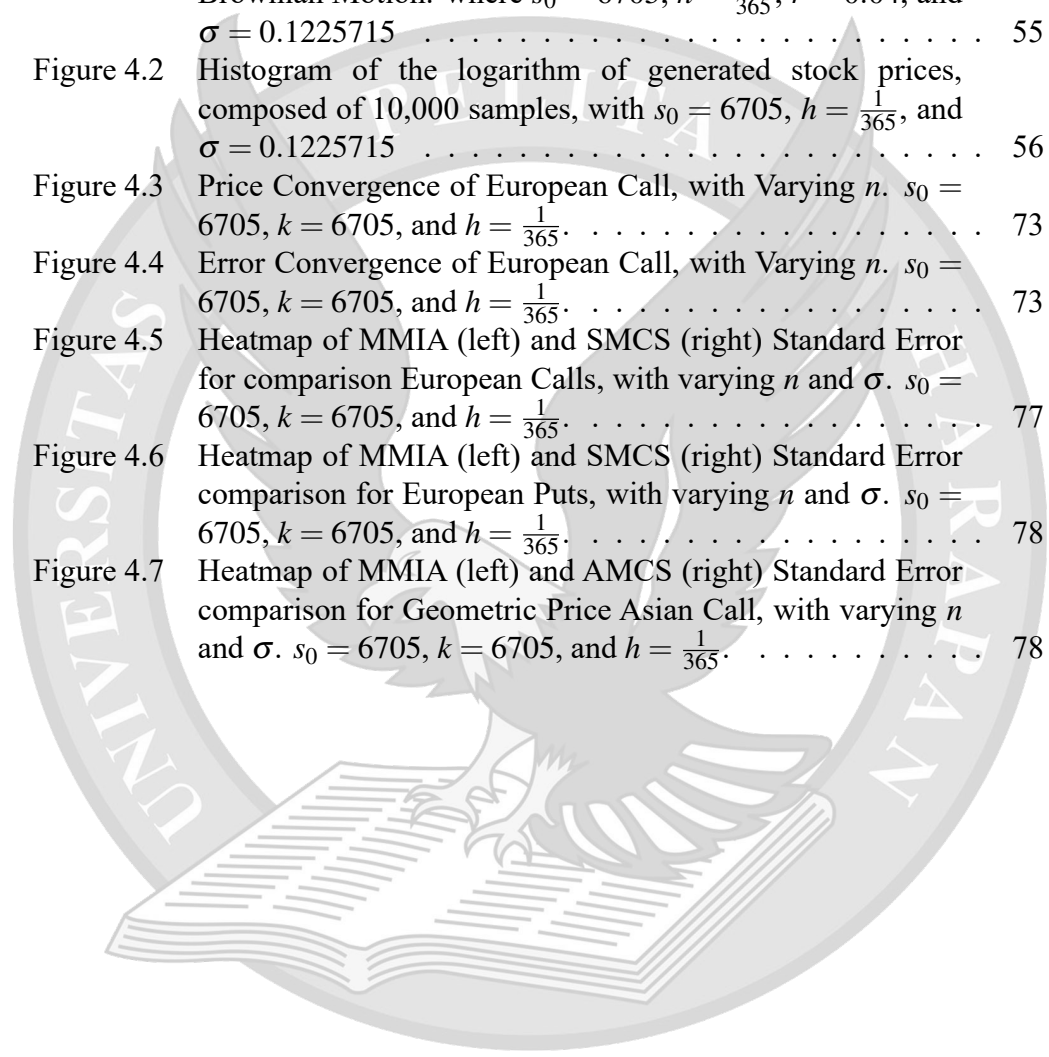
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